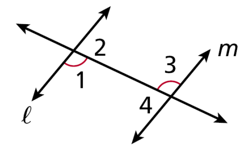
Attendance Problems. Justify each statement. (Explain why each statement is true.)

1. $\overline{QR} \cong \overline{QR}$

2. $l \parallel m$



Evaluate each expression for $x = 12$ and $y = 8.5$.

3. 2x + 7

4. 16x - 9

5. 8y + 5

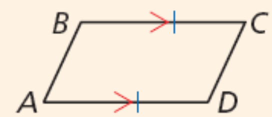
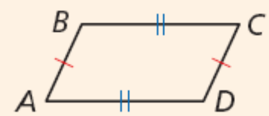
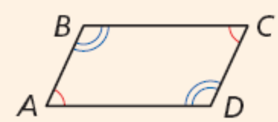
I can prove that a given quadrilateral is a parallelogram.

Common Core

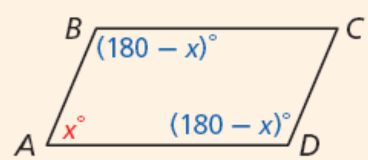
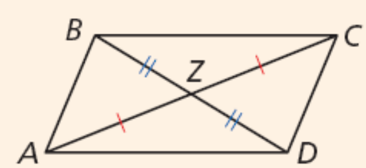
CC.9-12.G.CO.11 Prove theorems about parallelograms.
CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.
CC.9-12.G.MG.3 Apply geometric methods to solve design problems.

You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the other conditions shown.

Theorems — Conditions for Parallelograms

THEOREM		EXAMPLE
6-3-1	If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides ∥ and ≅ → ▱)	
6-3-2	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides ≅ → ▱)	
6-3-3	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. ∠s ≅ → ▱)	

Theorems — Conditions for Parallelograms

THEOREM		EXAMPLE
6-3-4	If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with ∠ supp. to cons. ∠s → ▱)	
6-3-5	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other → ▱)	

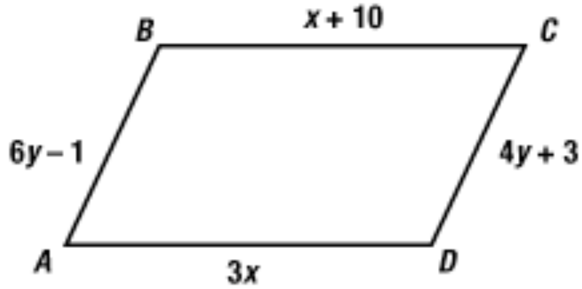
Question: Did you hear about the quadrilateral who wanted to be a parallelogram?
Answer: Yes, his opposite angles weren't quite up to par.

When it is incorrect, it is, at least *authoritatively* incorrect. -- <u>Hitchiker's Guide To The Galaxy</u>

Video Example 1.

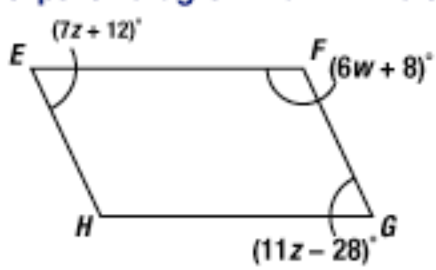
A.

Show that *ABCD* is a parallelogram for $x = 5$ and $y = 2$.

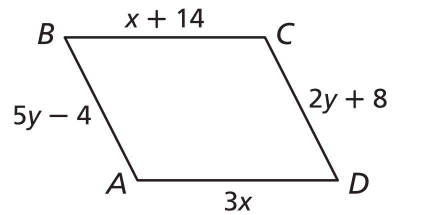


B.

Show that *EFGH* is a parallelogram for $z = 10$ and $w = 15$.



1 Verifying Figures are Parallelograms



A **Show that *ABCD* is a parallelogram for $x = 7$ and $y = 4$.**

Step 1 Find *BC* and *DA*.

$BC = x + 14$ — *Given* — $DA = 3x$

$BC = 7 + 14 = 21$ — *Substitute and simplify.* — $DA = 3x = 3(7) = 21$

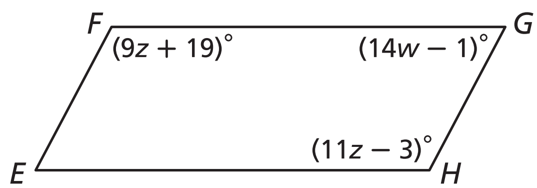
Step 2 Find *AB* and *CD*.

$AB = 5y - 4$ — *Given* — $CD = 2y + 8$

$AB = 5(4) - 4 = 16$ — *Substitute and simplify.* — $CD = 2(4) + 8 = 16$

Since $BC = DA$ and $AB = CD$, *ABCD* is a parallelogram by Theorem 6-3-2.

B **Show that *EFGH* is a parallelogram for $z = 11$ and $w = 4.5$.**



$m\angle F = (9z + 19)°$ — *Given*

$m\angle F = [9(11) + 19]° = 118°$ — *Substitute 11 for z and simplify.*

$m\angle H = (11z - 3)°$ — *Given*

$m\angle H = [11(11) - 3]° = 118°$ — *Substitute 11 for z and simplify.*

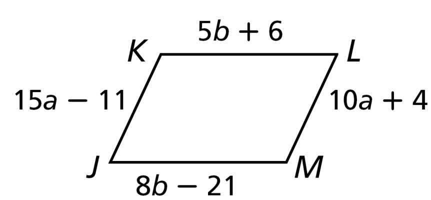
$m\angle G = (14w - 1)°$ — *Given*

$m\angle G = [14(4.5) - 1]° = 62°$ — *Substitute 4.5 for w and simplify.*

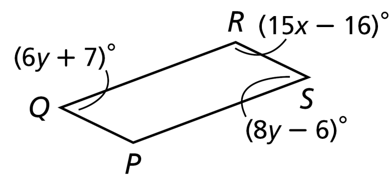
Since $118° + 62° = 180°$, $\angle G$ is supplementary to both $\angle F$ and $\angle H$. *EFGH* is a parallelogram by Theorem 6-3-4.

Example 1.

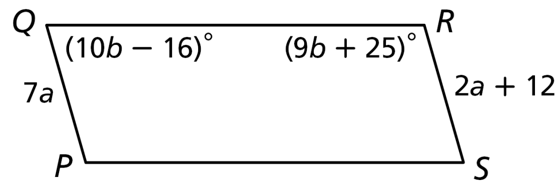
A. Show that JKLM is a parallelogram for $a = 3$ and $b = 9$.



B. Show that *PQRS* is a parallelogram for $x = 10$ and $y = 6.5$.

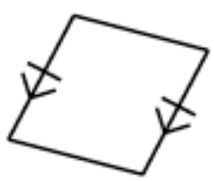


6. <u>Guided Practice</u>. Show that *PQRS* is a parallelogram for $a = 2.4$ and $b = 9$.

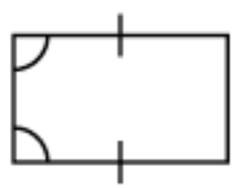


<u>Video example 2</u>. Determine if the quadrilateral must be a parallelogram. Justify your answer.

A.



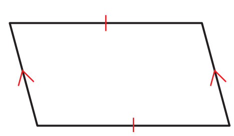
B.



2 Applying Conditions for Parallelograms

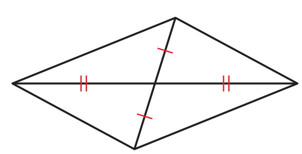
Determine if each quadrilateral must be a parallelogram. Justify your answer.

A



No. One pair of opposite sides are parallel. A different pair of opposite sides are congruent. The conditions for a parallelogram are not met.

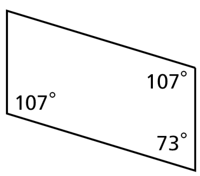
B



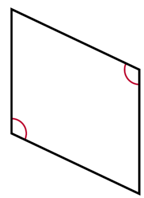
Yes. The diagonals bisect each other. By Theorem 6-3-5, the quadrilateral is a parallelogram.

<u>Example 2</u>. Determine if the quadrilateral must be a parallelogram. Justify your answer.

A.

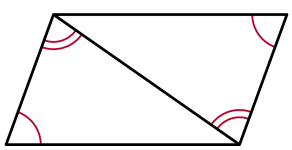


B.

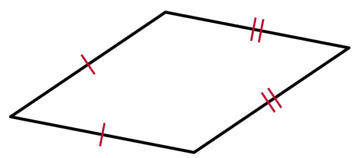


<u>Guided Practice</u>. Determine if the quadrilateral must be a parallelogram. Justify your answer.

7.

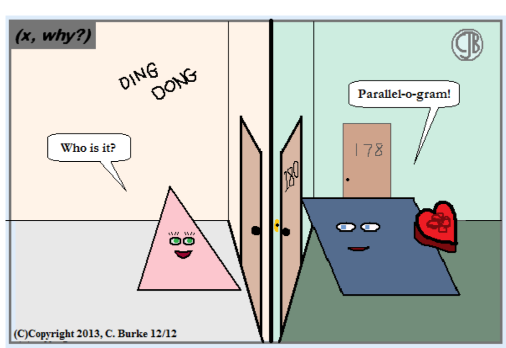


8.



Helpful Hint

To say that a quadrilateral is a parallelogram *by definition*, you must show that both pairs of opposite sides are parallel.

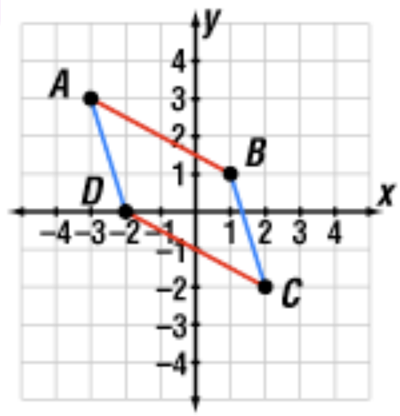


<u>Refer to video example 3</u>.

Show that the given quadrilateral is a parallelogram by using the given definition or theorem.

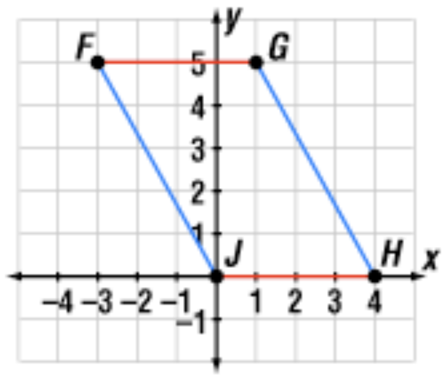
A.

$A(-3, 3)$, $B(1, 1)$, $C(2, -2)$, $D(-2, 0)$



B.

$F(-3, 5)$, $G(2, 5)$, $H(4, 0)$, $J(0, 0)$



3 Proving Parallelograms in the Coordinate Plane

Show that quadrilateral *ABCD* is a parallelogram by using the given definition or theorem.

A $A(-3, 2), B(-2, 7), C(2, 4), D(1, -1)$; **definition of parallelogram**

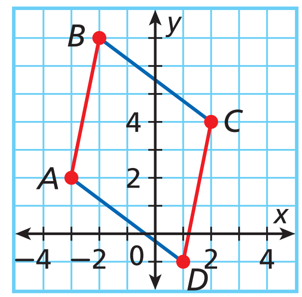
Find the slopes of both pairs of opposite sides.

$$\text{slope of } \overline{AB} = \frac{7-2}{-2-(-3)} = \frac{5}{1} = 5$$

$$\text{slope of } \overline{CD} = \frac{-1-4}{1-2} = \frac{-5}{-1} = 5$$

$$\text{slope of } \overline{BC} = \frac{4-7}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$$

$$\text{slope of } \overline{DA} = \frac{2-(-1)}{-3-1} = \frac{3}{-4} = -\frac{3}{4}$$



Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram by definition.

B $F(-4, -2), G(-2, 2), H(4, 3), J(2, -1)$; **Theorem 6-3-1**

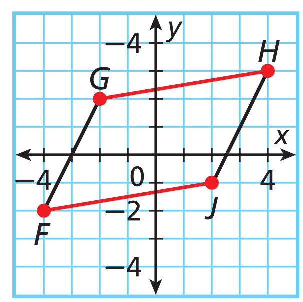
Find the slopes and lengths of one pair of opposite sides.

$$\text{slope of } \overline{GH} = \frac{3-2}{4-(-2)} = \frac{1}{6}$$

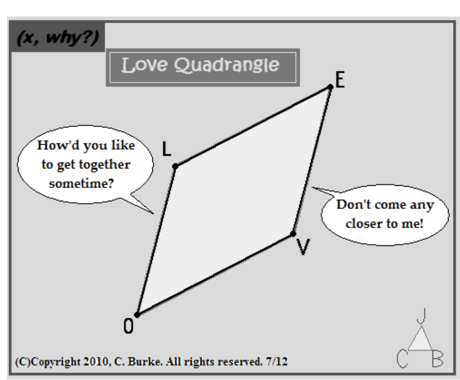
$$\text{slope of } \overline{JF} = \frac{-2-(-1)}{-4-2} = \frac{-1}{-6} = \frac{1}{6}$$

$$GH = \sqrt{[4-(-2)]^2 + (3-2)^2} = \sqrt{37}$$

$$JF = \sqrt{(-4-2)^2 + [-2-(-1)]^2} = \sqrt{37}$$

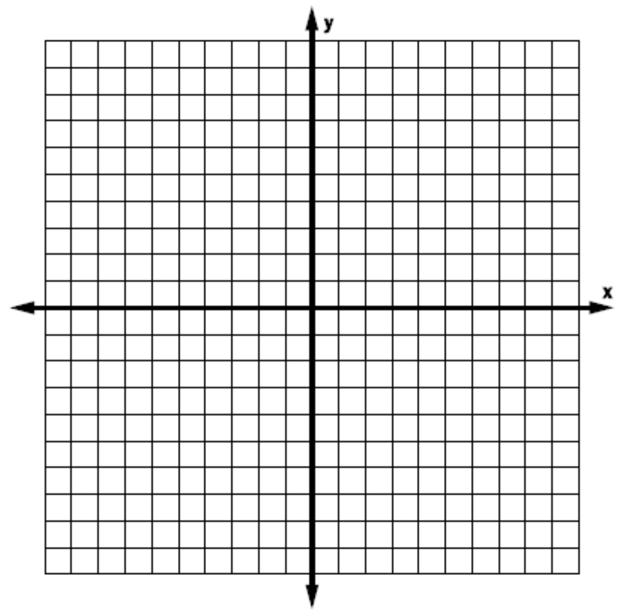


$\overline{GH}$ and $\overline{JF}$ have the same slope, so $\overline{GH} \parallel \overline{JF}$. Since $GH = JF$, $\overline{GH} \cong \overline{JF}$. So by Theorem 6-3-1, *FGHJ* is a parallelogram.

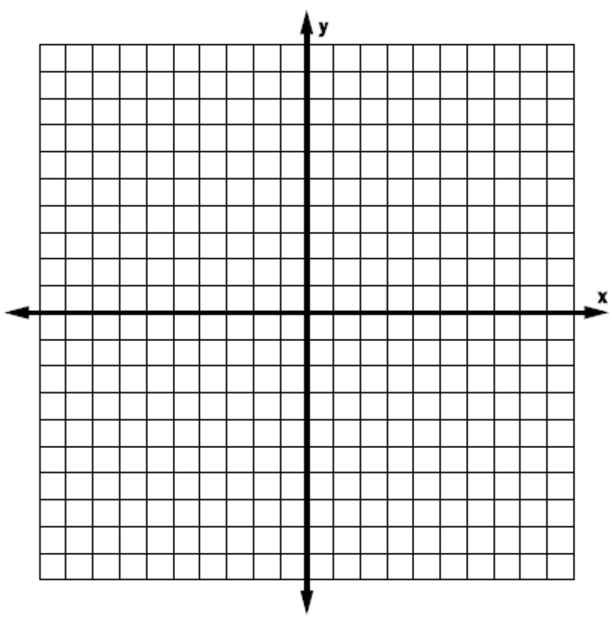


<u>Example 3</u>. Show that quadrilateral JKLM is a parallelogram by using the given definitions or theorems.

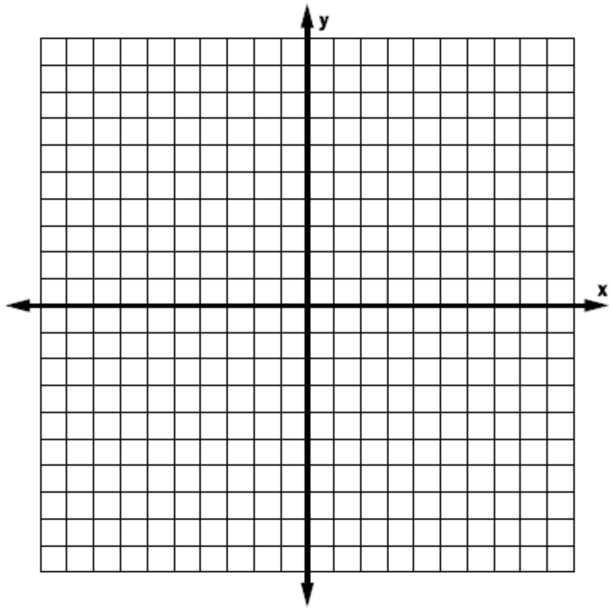
A. $J(-1, -6)$, $K(-4, -1)$, $L(4, 5)$, $M(7, 0)$.



B. $A(2, 3)$, $B(6, 2)$, $C(5, 0)$, $D(1, 1)$.



9. <u>Guided Practice</u>. Use the definition of a parallelogram to show that the quadrilateral with vertices *K*(–3, 0), *L*(–5, 7), *M*(3, 5), and *N*(5, –2) is a parallelogram.



Conditions for Parallelograms
Both pairs of opposite sides are parallel. (definition)
One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)
Both pairs of opposite sides are congruent. (Theorem 6-3-2)
Both pairs of opposite angles are congruent. (Theorem 6-3-3)
One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)
The diagonals bisect each other. (Theorem 6-3-5)

Helpful Hint

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

<u>6-3 Conditions for Parallelograms</u>

- *(p 414)* 9, 10, 11, 13-16, 22, 24-26, 32, 34.
- 6A Ready To Go On & posttests.